I. Adding Integers

by Kristi Carvajal

II. Grade Level: Math 7 (Advanced Math 6)

III. Length of Lesson: 5-6 days

IV. Overview

In this inquiry lesson, students will be investigating the question, *What is the relationship of the sum of the addends and their signs?* Students will use hands-on manipulatives, including two-colored counters, to add integers with different signs and to begin to discover rules for adding integers. These rules will include adding positive integers, adding negative integers, and adding positive and negative integers.

V. Context of the Lesson

This lesson should be taught at the beginning of the unit on addition, subtraction, multiplication, and division of integers. Through this weeklong inquiry lesson, students will discover rules for themselves that will prepare them well for further exploration of integer operations. They need to fully understand and be comfortable with integer addition before moving on to subtraction and the other operations, in order to avoid confusion and frustration.

The pre-assessment will help you determine the students' level of understanding of integers before the lesson. Use the results to teach a short pre-lesson on integers, focusing on integers in the real world. These real-life applications of integer addition will help students internalize knowledge of integers and prepare them for further lessons in the unit.

The pre-assessment results will also help you organize groups – consider putting students of similar abilities together in order to encourage collaboration, give more advanced students the opportunity to challenge themselves and each other, and give you the opportunity to give more focused guidance to groups that may be struggling.

This lesson uses a variety of adaptations for diverse learners (differentiation strategies): kinesthetic learners can use manipulatives, including two-colored counters, algebra tiles, and a mat, to physically add "integers"; visual learners can draw number lines and use colored digits instead of digits with a "+" or "-" sign; and auditory learners can listen to class and group discussions. Students who have trouble with abstraction in general have many opportunities to understand addition through discussion of real-world applications.

VI. Connections to State and National Standards

National Mathematics Standards for Grades 6-8 Number and Operations: In grades 6-8 all students should:

- develop meaning for integers and represent and compare quantities with them.

National Mathematics Standards for Grades 6-8 Algebra. In grades 6-8 all students should:

- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;

- relate and compare different forms of representation for a relationship.

Common Core State Standards:

- 6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

- 7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- 7.NS.A.1.B Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Virginia Standards of Learning (SOLs) for Mathematics:

- SOL 6.3 The student will
  a) identify and represent integers;
  b) order and compare integers.

- SOL 7.3b The student will add, subtract, multiply, and divide integers.

VII. Unit Goals and Lesson Objectives

a. Know (facts)
   - Integer definition
   - Rational numbers
   - Positive and negative signs
   - Zero pair
   - Two-colored counters
   - Rules for adding integers
   - Whole numbers
   - Opposites
   - Integers in the real world (examples: temperature changes; sports; checking account; altitude)

b. Understand (big idea)

   Integer operations follow patterns that we can recognize and use to formulate rules.

c. Do (skills)

   - Find the sum of two positive integers.
   - Find the sum of two negative integers.
   - Find the sum of a positive and negative integer.
   - Draw an image of a real-life integer problem.
   - Mathematically solve a real-life integer addition word problem.

VIII. Pre-assessment of students' prior knowledge and/or skills

Give the pre-assessment (see Resources) three days before beginning this inquiry lesson. It will give you a sense of students' background knowledge of integers and what basic information on integers you will need to teach before beginning the unit on addition.

The results from this pre-assessment will help you determine how much instruction about integers you will need to give before beginning the lesson on integer addition. Some information you may need to clarify is: comparing integers; placing and recognizing integers on a number line; and describing how we use integers in the real world.

Use the pre-assessment results as well to group students in groups of like ability and knowledge. This will encourage collaboration, give advanced students the opportunity to challenge themselves and each other, and allow you to give more focused guidance to groups that may be struggling.

IX. Materials

Pre-assessment:

- Pre-assessment paper/pencil worksheet
- Pencils
- Colored pencils/markers

Structured Inquiry Lesson:

- Pencils
- Chart/whiteboard
- Two-colored counters
- Colored pencils/markers
- Large construction paper
- Teacher notebook
- Integer Direct Instruction handout
- KWL Chart
- Integer Addition Worksheet
- Day 2 Exit Ticket
- Integer Extension Phase Questions handout (cut these into individual slips)
- Structured Inquiry Presentation Checklist – Adding Integers
- Journal Writing handout
- Adding Integers Journal
- Integer Word Problems Exit Ticket

Post-assessment:

- Post-assessment paper/pencil worksheet
- Pencils
- Colored pencils/markers

X. Level of Inquiry: Structured

This is a structured inquiry lesson. Students will be given the question for investigation and a procedure to follow. They will carry out the procedure in small groups, using given materials, and recording their data on a worksheet. They will then present their findings to the class. Although this is a very

structured lesson, students will still be investigating patterns and discovering deeper relationships among numbers for themselves. You should act as a facilitator rather than an instructor and ensure that your students are actively engaged.

XI. Teaching Strategies

Pre-Assessment
Give this at least 3 days before this lesson, to give you time to teach a brief introductory lesson on integers, covering especially those areas that students missed on the pre-assessment. Use this introductory lesson only to make sure your students understand what integers are, NOT to introduce addition.

Day 1
After the pre-assessment and before beginning the inquiry lesson on integer addition, give an introductory lesson on integers. Give students a KWL chart (see Resources) and ask them to write down what they Know about integers and what they Want to know. Discuss what students know and want to know about integers, then pass out the “Integer Direct Instruction” worksheet to review comparing integers, ordering integers, and identifying integers on a number line.

Introduce students as well to the two-colored counters, explaining that the yellow/white side represents positive and the red side represents negative. Explain zero pairs, and guide the discussion by inviting students to start thinking about questions that occur to them about integers. Have students keep the KWL charts, so they can write what they have Learned after the inquiry lesson.

Before the end of class, hand out the “Adding Integers Journal” worksheet (see Resources) to get students thinking about and articulating what they believe a zero pair is and how it might be use in adding integers.

Day 2
At the beginning of class, hold a class discussion of the “Adding Integers Journal” worksheet to give students an opportunity to articulate their thoughts about zero pairs. After they share their thoughts, go over the definition of zero pairs using the "Integer Direct Instruction" worksheet from yesterday. This will lead the class into today's lesson about adding integers using zero pairs.

Before dividing the class into groups, review the information from the KWL chart from the Day 1.

Then write the investigation question on the board: *What is the relationship of the sum of the addends and their signs?* After introducing the question, divide the class into groups and invite groups to get supplies and begin the lesson. Each group will need two-colored counters, pencils, the “Integer Addition” worksheet, and colored pencils or markers. Let students know that you will be reading and correcting the worksheets, but not grading them. Tell the students to work on the integer addition problems in the handout in three ways: by using their two-colored counters; by drawing the numbers in color (red is negative; yellow is positive); and by solving them mathematically. Monitor the groups to make sure that all the students are participating in the group discussions, that all the students are drawing pictures of the integers on their papers, and that all the students are modeling integer addition problems using their two-colored counters. Make sure to listen in on student discussions about adding integers: notice any patterns they are discovering, ask guiding questions if they are confused, and clear up any misconceptions. Jot down notes as you circulate.

After they are finished, have the students hand in their papers so you can check them for accuracy. Do not grade these papers, but use them as a formative assessment of student learning. Hand out the Day 2 Exit Tickets, and ask students to answer one of the three on the handout.

Day 3

Hold a brief discussion about the activity from Day 2. Invite students to share their responses from the Exit Ticket. Divide the class into groups again, with the investigation question still on the board. Have a bowl ready with slips of paper, each of which has one "Extension Phase" question on it. Have a volunteer from each group pull a slip of paper from the bowl; this will be the real-world integer addition problem that that group will investigate. In this way, no two groups will have the exact same question, but all students will solve similar integer addition word problems. In addition, hand out the "Structured Inquiry Presentation Checklist" so that students know what they need to include in their presentations and how they will be assessed.

Ask students to solve the problem in the "Extension Phase" question both pictorially and mathematically. Each group will determine, through discussion, how they want to solve the problem pictorially: do not give students any specific suggestions, but encourage them to come up with their own ideas.

Once they are finished solving their problems, have them present their findings to the class. Ask each group to read their real-world problem aloud, then explain how they solved it pictorially and mathematically. Use the "Structured Inquiry Presentation Checklist" as a formative assessment to note whether they have covered all criteria. Invite the students not presenting to ask questions of the presenters.

Day 4

Finish up any group presentations that you didn't have time for on Day 3. Once all the presentations are done, discuss their "Structured Inquiry Presentation Checklist" with each group individually.

Pass out the "Journal Writing" handout. Have each group choose a presentation by a different group, and discuss that presentation's similarities to and differences from their own presentation, using the handout as a jumping-off point for their discussion.

Hold a class discussion about the presentations and the different ways to work through integer addition problems.

For an extension assignment and to see how students can solve integer addition word problems individually, use the worksheet "Integer Word Problems Exit Ticket" (see Resources). Students will solve two of the three problems mathematically and/or pictorially.

Day 5

At the beginning of class, go over the answers to all three word problems from the "Integer Word Problems Exit Ticket."

Give students the post-assessment (see Resources, one per student).

Once they have finished with the post-assessments, ask them to take out their KWL charts again and fill in the last section, about what they have learned. Hold a whole-class discussion about what they have learned. Ask questions specifically about larger rules they may have discovered. See if they can hypothesize about rules for subtraction, and solicit questions about multiplying and dividing integers. If you have time, discuss some more real-world examples of integer addition, and see if you can bring

out integer subtraction in these situations as well. See if you can get a sense of student reactions to the method of teaching, as well as what they have learned.

XII. Assessment Plan

As stated in section VII, the goals for this structured inquiry lesson include knowledge about integers abstractly and in real life, adding integers of the same and different signs, and discovering and applying patterns as rules to integers. Assessment of these objectives is accomplished in various ways.

Summative assessments include the pre- and post-assessments; comparison of these will give you a sense of how much the students have learned about integers and integer addition rules. This will help you as you plan for your lesson on subtracting integers, and the rest of your unit on integers.

Formative assessments include the "Addition Integer" worksheet, the "Structured Inquiry Presentation Checklist," the Exit Tickets, and the "Journal Writing" papers, as well as your notes from observation of group discussion and activities. The multiple opportunities for formative assessment will give you a good sense of how the students are learning; the "Journal Writing" is also an opportunity to assess students' facility with and understanding of data presentation.

XIII. Resources

Books:
Llewellyn, D. (2007). *Inquire Within: Implementing Inquiry-Based Science Standards in Grades 3-8*, 2nd Edition. Corwin Press.

Bell, R. L., Smetana, L., and Binns, I. (October 2005). "Simplifying Inquiry Instruction." *The Science Teacher*, 30–33.

Adding Integers
Structured Inquiry Pre-Assessment

1. **In your own words, define what an "integer" is *and* give a numerical example of an integer.**

_____
_____
_____.

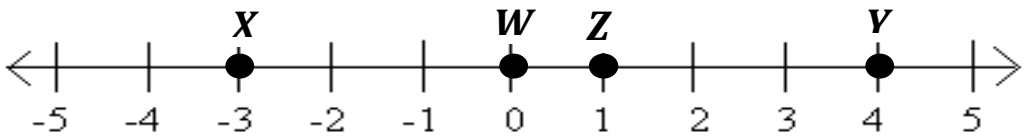
2. **List two situations in which integers are used in the real-world today.**

   A) _____

   B) _____

3. **Use the number line to answer the question below.**

   Which point on the number line represents the least integer?



   a) Letter W
   b) Letter X
   c) Letter Y
   d) Letter Z

4. **Directions:** Circle the answers you want to select. You must select all correct answers.

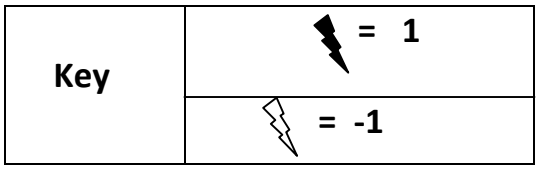
   Which of the examples of comparing integers below is true? Circle all the correct boxes comparing integers.

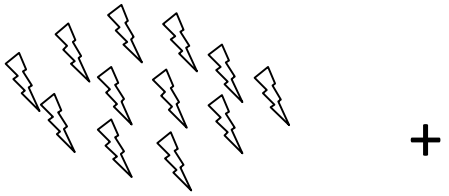
-4 > -16	**-25 = 25**	**0 > -9**
-2 < 2	**-8 < -3**	**-5 > 15**

5. **<u>Directions:</u> Solve the problem below. Write your answer in the box.**

-8 + 3 =

6. **<u>Directions:</u>** Use the key below to answer the next question. Write your answer in the box.





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7. **SKATEBOARDING. Hakeem starts at the bottom of a half pipe 6 feet below street level. He rises 14 feet at the top of his kick turn. Where is Hakeem?**

A 8 B -8 C 20 D -20

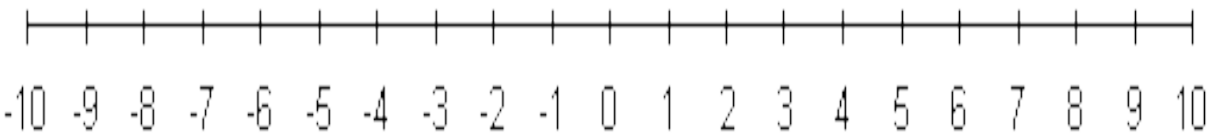
8. **Lou asks Joyce to borrow $6.00 on Monday, and another $5.00 from her on Wednesday. If Lou pays back $8.00 on Friday, how much does he still owe Joyce?**

a) $1.00 b) $3.00 c) $9.00 d) $19.00

9. **Directions:** After showing your thinking, write your answer in the box.

What is the value of -27 + 13?

Directions: Use the number line below to answer the next question.



10. Which group of numbers is ordered from least to greatest?

a) -6, -9, 3, 0, 4
b) 4, 3, 0, -6, -9
c) 0, -9, -6, 3, 4
d) -9, -6, 0, 3, 4

Structured Inquiry Adding Integers Journal

1. What do you think a zero pair is?

_____

_____

_____

_____

2. How do you think a zero pair is used with integers?

_____

_____

_____

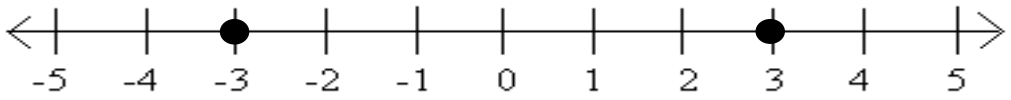
_____

3. Can you draw a picture to represent your thinking of using a zero pair?

Structured Inquiry Integer Direct Instruction

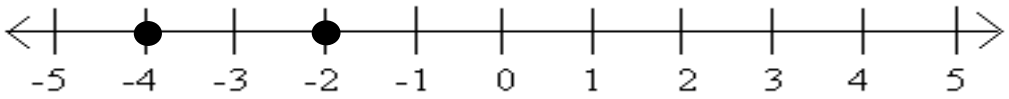
Integers are the set of whole numbers, their opposites, and zero.

- Positive integers are _____ zero.
- Negative integers are _____ zero.
- Zero is neither positive nor negative.



- An integer and its opposite are the same distance from zero on a number line.
  The opposite of -3 = ____
- A negative integer is always _____ a positive integer.
- When comparing *two negative integers,* the negative integer _____ to zero is greater.

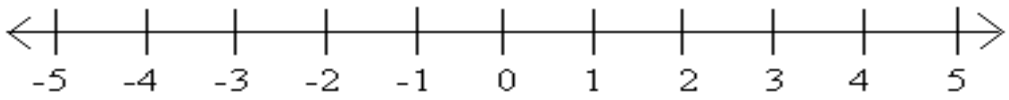
  Example:



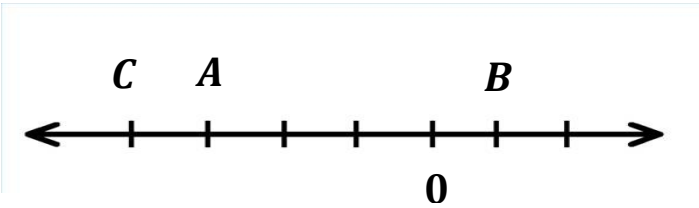
-4 ____ -2

Representing Integers on a Number Line:

Directions: Graph -4 and its opposite on the number line. Draw an arrow to identify the larger integer.



Directions: Identify the integer represented by each letter on the number line.



Letter A: ___ Letter B: ___ Letter C:___

Compare and Order Integers:

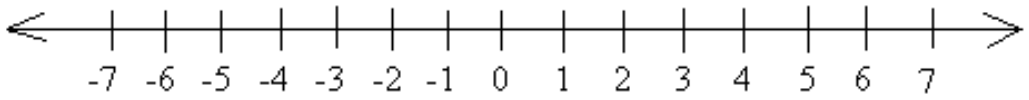
Directions: Write your answer in the box.

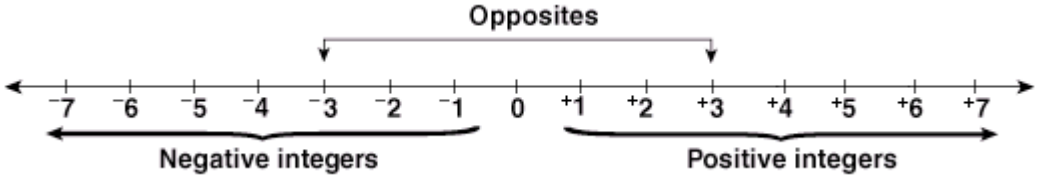
Compare the integers using the correct mathematical symbols { <, >, =}.

a) -6 ☐ -8 b) 0 ☐ -15 c) -7 ☐ 7

d) 2 ☐ -1 e) -5 ☐ 5 f) 4 ☐ + 4

Directions: Use the number line below to order the following numbers from least to greatest. [-3, 6, 0, -8, -5]





Zero Pair: _____.

- A ☐ and a ☐

are called a ☐ .

⊖ = ⊕ =

Example:

⊕ ⊕ + ⊖ ⊖ = 0

KWL Chart

What I Know	What I Want to Know	What I Learned

Integer Addition Worksheet

 = (–1)  = (+1)

Zero Pair: where a positive sign and a negative sign = 0.

Directions: Model the addition problems below using two-colored counters. Then, draw and solve the integer addition problems below using positive and/or negative integer symbols.

1) 3 + 4 =

2) 5 + 3 =

3) 7 + 2 =

4) 2 + 1 =

5) If there were a rule, trick, tip, or an easy way to remember how to add two positive integers, what would you tell someone? _____
_____
_____
_____

Directions: Model the addition problems below using two-colored counters. Then, draw and solve the integer addition problems below using positive and/or negative integer symbols.

6) -5 + (-4) =

7) -3 + (-2) =

8) -13 + (-1) =

9) -7 + (-3) =

(Integer Addition worksheet, page 2)

10) Pretend you are the teacher, and you will be teaching another group of students how to add two negative integers. What would you tell that group of students to help them add two negative integers? _____

_____

_____

_____

Directions: Model the addition problems below using two-colored counters. Then, draw and solve the integer addition problems below using positive and/or negative integer symbols.

11) -7 + 3 =

12) 9 + (-4) =

13) -2 + 5 =

14) 10 + (-5) =

15) -6 + 0 =

16) -4 + 4 =

17) You are trying to teach your best friend how to add integers with two different signs. What would you tell him/her?

_____

_____

_____

18) Work in your group and make up your own integer addition problem with two different signs. Show your understanding by drawing a picture and solving the problem.

Day 2 Exit Ticket

Directions: Choose ***one*** of the three choices below. Write your thoughts using complete sentences.

1. Two things I learned today are …

_____
_____
_____
_____

2. By completing this investigation, I now know…

_____
_____
_____
_____

3. Connections I made during this lesson include…

_____
_____
_____
_____

Integer Extension Phase Questions

1) The change in the number of students enrolled at a school over three months is shown in the following table.

School Enrollment

Month	Change
October	-14
November	7
December	3

The number of students enrolled at the end of September was 200. What was the number of students enrolled in the school at the end of December?

2) A whale is 42 feet below the surface of the water. She rises 25 feet and finally rises another 16 feet. If there are no other changes, where is the whale's location?

3) A submarine is 40 feet below the surface of the water. The sub rises 28 feet, goes down 7 feet and finally rises another 17 feet. If there are no other changes, where is the submarine's location?

4) Ginny lost 53 pounds. She gained 12 pounds back and then lost an additional 16 pounds. What integer represents Ginny's change in weight?

5) HOT AIR BALLOON. A hot air balloon rises 36 feet into the air. It then descends 17 feet and finally rises another 6 feet. If there are no other changes, where is the balloon's location?

6) Billy purchased a stock from a company for $40.00. The next day the stock rose $5.00. On the second day, the stock dropped $10.00, and on the third day, the stock dropped another $7.00. On the fourth day, the stock rose $8.00. How much was the stock worth at the end of the fourth day?

7) A scuba diver dives 25 feet under the surface of the ocean. He changes his position by coming up 11 feet. He finally decides to go down another 13 feet. After these changes, what is the diver's location?

8) During winter, the temperature at 6:00 a.m. in North Dakota was 9 °F. During the day, the temperature rose 12 degrees. At 5:00 p.m. the temperature dropped 18 degrees. What was the temperature after these changes at 5:00 p.m.?

9) Hank recorded the outside temperature of his classroom for a week.

Outside Temperature

Day	Temperature (° F)
Monday	19 °
Tuesday	-10 °
Wednesday	7 °
Thursday	2 °
Friday	-8 °

What integer represents the changes in temperature by Friday?

10) An elevator can travel several levels below ground. Its first stop is four levels down from the ground floor. It then travels nine levels above the first stop. For the final stop, the elevator then travels three floors down from the second stop? If there are no other changes, where is the elevator's location after the final stop?

Structured Inquiry Presentation Checklist- Adding Integers

	Yes	No
Time Management		
• Did I use my time wisely during the presentation?		
Organization		
• Did I evenly space the question, illustration, and math work on the poster?		
• Does the math work clearly answer the question?		
Audience		
• Did each member of the group present in a clear, even tone?		
• Did each member make eye contact with the audience?		
Presentation		
• Was the poster neatly colored?		
• Was the question, illustration, and math work clearly visible to the audience?		
• Did I clearly explain how the illustration correlates to the problem?		
Comments		

Journal Writing
Structured Inquiry Lesson- Adding Integers

Directions: Get into your group. Pick one group's presentation and discuss and write the similarities and differences of their project to yours. Remember, your Journal Writing uses complete sentences.

Some things you may consider discussing and writing about are:

- How is their illustration similar/different from our illustration?
- Is there another way our group could have illustrated our question differently? If so, explain.
- How could our group have made our poster more presentable when compared to another group's poster?
- When we solved our question mathematically, how was it similar to or different from the other group?
- Is there another way to solve an addition integer question mathematically?

Adding Integers Word Problems Exit Ticket

Directions: Choose ***two*** of the three choices below. Show your work either mathematically or pictorially to answer the questions below.

1. **Directions: Solve the problem below either mathematically or pictorially. Write your answer in the box.**

   FOOTBALL. A team gained 5 yards on their first play of the game. Then they lost 6 yards. On the third down, they lost another 3 yards. What is the total change in yardage?

2. **A ray moves 80 feet below the surface of the ocean. It rises 43 feet and dives further into the water 12 feet. It finally rises 18 feet. The final location of the ray is ---**
   a) 93 feet above the surface
   b) 93 feet below the surface
   c) 31 feet above the surface
   d) 31 feet below the surface

3. **The adjustments in the amount of money in a checking account over the last three months is shown in the following table.**

Checking Account

Day	Money ($)
January	22
February	-6
March	-40

   The amount of money in the checking account at the end of December was $200. What was the amount of money in the checking account at the end of March?

   A $165 B $176 C $182 D $196

Adding Integers
Structured Inquiry Post Assessment

1. **In your own words, define what an "integer" is *and* give a numerical example of an integer.**

_____
_____
_____
_____

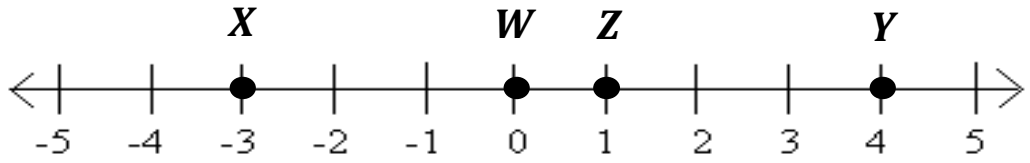
2. **List two situations in which integers are used in the real-world today.**

   A) _____

   B) _____

3. **Use the number line to answer the question below.**

   Which point on the number line represents the least integer?



a) Letter W
b) Letter X
c) Letter Y
d) Letter Z

4. **Directions:** Circle the answers you want to select. You must select all correct answers.

   Which of the examples of comparing integers below is true? Circle all the correct boxes comparing integers.

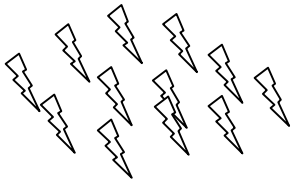
-4 > -16	**-25 = 25**	**0 > -9**
-2 < 2	**-8 < -3**	**-5 > 15**

5. **Directions: Solve the problem below. Write your answer in the box.**

   -8 + 3 = ☐

6. **Directions:** Use the key below to answer the next question. Write your answer in the box.

Key	
	= 1
	= -1

 +  ☐

7. **SKATEBOARDING. Hakeem starts at the bottom of a half pipe 6 feet below street level. He rises 14 feet at the top of his kick turn. Where is Hakeem?**

   A 8 B -8 C 20 D -20

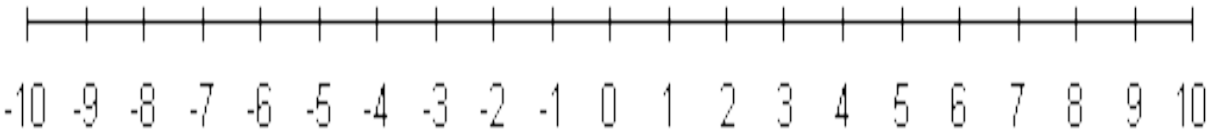
8. **Lou asks Joyce to borrow $6.00 on Monday, and another $5.00 from her on Wednesday. If Lou pays back $8.00 on Friday, how much does he still owe Joyce?**

b) $1.00 b) $3.00 c) $9.00 d) $19.00

9. **Directions:** After showing your thinking, write your answer in the box.

What is the value of -27 + 13?

Directions: Use the number line below to answer the next question.



e) -6, -9, 3, 0, 4
f) 4, 3, 0, -6, -9
g) 0, -9, -6, 3, 4
h) -9, -6, 0, 3, 4