I. Slope of Parallel and Perpendicular Lines

by Ann Swanberg

II. Grade Level: Math 8 (Geometry)

III. Length of Lesson: 4 days

IV. Overview

In this inquiry lesson, students will be investigating the questions, *What is the relationship between the slopes of two perpendicular lines?*; *What is the relationship between the slopes of two parallel lines?*; and *What is the relationship between the slopes of two intersecting lines that are not perpendicular?* Students will take pictures of parallel, perpendicular, and intersecting lines in the real world; they will draw parallel, perpendicular, and intersecting lines on coordinate planes; they will calculate slopes; and they will discover how the slopes of each set of lines relate to each other, generalizing to a conclusion about slope relationships. This is a structured inquiry lesson, so students are given the questions for investigation and the experimental procedure, but they will carry out the experiments and come to data-driven conclusions on their own.

V. Context of the Lesson

This lesson is best taught in a unit on graphing, just before lessons on symmetry and transformations. Before this lesson, students should have just learned the formulas for midpoint, slope, and distance, as well as how to determine whether lines are parallel, perpendicular, or neither. This lesson applies their knowledge of slope to an investigation of parallel, perpendicular, and intersecting lines.

Before beginning this lesson on slope relationships, students must know how to calculate slope. Although they should have learned this formula in algebra already, many may still struggle with it: assess your students' readiness with a pre-assessment (see Resources). Some misconceptions students may have about slope are: they compute the change in x over the change in y; they add instead of subtract; and they forget the relationship between the slopes of perpendicular lines.

Use results from the pre-assessment to group students of similar skill levels together, to encourage collaboration, enable advanced students to challenge themselves further, and give you the opportunity to offer further guidance to groups that are struggling. This lesson also uses a variety of adaptations for diverse learners (differentiation strategies): kinesthetic learners can trace the lines they find in their "scavenger hunts" with their hands; visual learners can draw the lines; and auditory learners can listen to class and group discussion.

VI. Connections to State and National Standards

National Mathematics Standards for Grades 6-8 Geometry: In grades 6-8 all students should

- precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
- use visual tools such as networks to represent and solve problems;
- recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

Virginia Standards of Learning (SOLs) for Mathematics:

- SOL G.3 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
  a) investigating and using formulas for finding distance, midpoint, and slope;
  b) applying slope to verify and determine whether lines are parallel or perpendicular.

- SOL G.4 The student will construct and justify the constructions of
  c) a perpendicular to a given line from a point not on the line;
  d) a perpendicular to a given line at a given point on the line;
  g) a line parallel to a given line through a point not on the given line.

- SOL 8.14 The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.

- SOL 8.15 The student will graph a linear equation in two variables.

Common Core State Standards:

- HSG.GPE.B.4 Use coordinates to prove simple geometric theorems algebraically.

- HSG.GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

VII. Unit Goals and Lesson Objectives

a. Know (facts)
   - Parallel
   - Perpendicular
   - Slope
   - Reciprocal
   - Negative reciprocal
   - Vertical and horizontal
   - Rise and run
   - Coordinate grid/coordinate plane
   - Quadrants
   - X-axis and y-axis
   - Ordered pairs

b. Understand (big idea)

   Comparing slopes of lines can tell you about their relationship (perpendicular, parallel, or neither).

c. Do (skills)

   - Calculate slope
   - Graph and analyze lines
   - Compare lines using slope

- Find the equation of a line parallel or perpendicular to a given line that passes through a given point
- Use technology to create a presentation about line interactions

VIII. Pre-assessment of students' prior knowledge and/or skills

Students should be able to calculate slope and to understand and analyze graphed lines before they can compare slopes of lines. The results from the pre-assessment (see Resources) will help you determine how much instruction about calculating slope and using graphs you need to give before this inquiry lesson. It will also give you a baseline of student understanding before the lesson that you can then compare to results of your post-assessment.

Group students of similar abilities together. This way, you can offer more focused guidance to those groups who are struggling most with slope calculations.

IX. Materials

Pre-assessment:
- Pre-assessment paper/pencil worksheet
- Pencils

Structured Inquiry Lesson:
- Cameras
- Scavenger Hunt handout
- Slope Inquiry Data Chart handout
- Lines for Differentiation handout
- Checklist for Discovering Line Relationship teacher rubric
- Grid for Pictures handout
- Peer Evaluation Form handout
- Slopes Presentation Rubric
- Slope Anchor Activity (optional homework)
- Bulletin board paper and other materials for poster presentations
- Markers
- Coordinate graph paper
- Computers
- Rulers
- Calculators
- Geometer's sketchpad
- Protractors
- Index cards

Post-assessment:
- Post-assessment paper/pencil worksheet
- Pencils

X. Level of Inquiry: Structured

This is a structured inquiry lesson. You will present the students with the questions for investigation as well as the experimental procedure. Students will be responsible for collecting data and reaching a conclusion. Although this is a very structured lesson, the approach is still math by inquiry: rather than plugging numbers into received equations, students will be noticing patterns and discovering deeper

relationships among numbers. As they investigate, act as a facilitator and ask questions to get them thinking on their own about how to interpret the data they are collecting.

XI. Teaching Strategies

Pre-Assessment
Give this a day or two before giving this structured inquiry lesson on slope. This will give you time to determine groups, as well as to plan any guidance you may feel it necessary to give about slope calculation and graphing.

Day 1
As a way to introduce slope and to increase students' awareness of slope as an important feature of a line, start the lesson with a "scavenger hunt" for lines of particular slopes in the real world (see Resources for handout). Separate students into the groups you have planned based on pre-assessment results, and give each group a camera. Give them 15-20 minutes to take pictures of parallel, perpendicular, and intersecting lines in and around the school. Ask them specifically to take pictures of stairs as part of their presentation. When they return, hold a class discussion on the implications of parallel and perpendicular lines, and ask how things would be different of the parallel and perpendicular lines they found were changed. Draw students' attention to the steps they photographed. Use the words "rise" for the height of each step and "run" for its length. Discuss steepness of stairs in terms of rise over run. Solicit student ideas about how to calculate steepness, or slope, and guide the discussion into abstraction of the idea of "rise over run," the slope formula, $(y – y) / (x – x)$.

Then introduce the inquiry lesson. Write the questions for investigation on the board:

What is the relationship between the slopes of two perpendicular lines?

What is the relationship between the slopes of two parallel lines?

What is the relationship between the slopes of two intersecting lines that are not perpendicular?

Invite students to discuss these questions in their groups and, without looking at graphs or making calculations, to come up with hypotheses for each one. As they discuss, circulate through the room and jot down notes about their discussions, as a sort of formative pre-assessment both of their understanding of slope and of their comfort level with inquiry-based learning.

Tell them that at the close of the lesson (in three days), they will present their findings to the class, in 3-5 minute presentations. The first part of their presentations should include their pictures and predictions about the relationships between the lines. Students can decide as a group how they would like to present their data.

Day 2
Before dividing the class into groups, hold a brief class discussion of slope. See how much students remember of the slope formula they derived in class on Day 1, and discuss possible values of slope (negative, positive, zero, and undefined/no slope). Solicit student ideas, and guide the class into understanding of slope as: positive if the line rises, or goes uphill; negative if the line falls, or goes downhill; zero if the line is flat, or horizontal; and no slope/undefined if the line is up and down, or vertical.

Have students form the same groups as on Day 1, and pass out to each group: twelve coordinate planes, a data recording sheet (see "Slope Inquiry Data Chart" in Resources), and protractors. Have groups draw four sets of parallel lines, four sets of perpendicular lines, and four sets of intersecting lines. Be sure to explain that for perpendicular lines, one set should include a horizontal and vertical line. As they work, monitor them to make sure their lines are parallel and perpendicular. If they are having trouble, ask them to describe the steps they follow when they draw lines. If you need to, give them samples with the lines already drawn as a visual aid (see "Lines for Differentiation" in Resources). Jot down notes as you circulate through the class about the students' understanding of slope, facility with graphing, and attitudes about inquiry.

Once they have drawn their lines, have them compute the slopes of the all the lines, enter this data in the data chart, and complete the chart. They should find that slopes of parallel lines are equivalent; that slopes of perpendicular lines have a product of -1; and that slopes of intersecting lines have no consistent relationship. Have them discuss their findings in their groups. You may use the "Checklist for Discovering Line Relationship" (see Resources) as a way to record student findings. Have a class discussion on slope relationships to make sure everyone is on the same page and to discuss errors.

Day 3

Have students revisit the photographs they took on Day 1 and have them use Microsoft Publisher to overlay a coordinate grid (see "Grid for pictures" in Resources) on the photos they took on day 1. Have them draw lines, find slopes, and use the rules they discovered to prove their lines are parallel, perpendicular, or both. Circulate through the classroom asking guiding questions and jotting down notes about their understanding of slope, facility with graphing, and attitudes about inquiry.

Give the groups time to work on their presentations. Remind them that they should include: pictures of all line relationships discussed; their twelve coordinate planes with lines used to develop the rules for proving that lines are parallel, perpendicular, or neither; pictures with coordinate grids overlaid; slopes of the lines in their pictures; evidence that they have used the rules they discovered to prove the relationships between their lines; at least three sample questions for the class; and a creative, data-driven summary of their findings.

For those groups that finish early, and/or as a homework assignment, pass out "Parallel and Perpendicular Lines" handouts (see Resources). These have lines and points marked on them; have students give an integral point that would be on a line that passes through the point and is perpendicular, as well as one for a parallel line that passes through the given point. For homework, have them write these points on index cards.

Day 4

Separate students into their groups again. Have them check their homework against each other by plotting their points on the graph paper and making sure all points on the imagined parallel line are in a line with each other, and likewise for the imagined perpendicular line. Collect their index cards and the completed graph paper.

Hold class presentations. Have students fill out peer evaluation checklists for their classmates' presentations, while you fill out a presentation rubric (see Resources for both).

After the presentations, discuss the results as a class – did all the groups find the same thing? Did they express it the same way (for example, the relationship of the slopes of the perpendicular lines could be expressed as a product of -1 or a negative reciprocal)? Did they present their results in the same way? If there was variation, discuss data presentation. If there was error, discuss possible sources for it.

The next day, administer the post-assessment. See Resources for additional homework or in-class worksheets if you would like to give your students more practice with slope.

XII. Assessment Plan

As stated in section VII, the goals for this structured inquiry lesson include understanding how the slopes of lines determine how two lines relate to each other, and developing graphing and calculation skills. Assessment of these learning goals is accomplished through multiple measures.

Summative assessments include the pre- and post-assessment. The pre-assessment will help you determine your students' level of knowledge about slope and their facility with graphing. With some nearly identical questions, the post-test measures the same things, so a comparison of the two will give you a good sense of how successful this lesson was at reaching the learning goals.

Formative assessments include the students' "Slope Inquiry" data charts; your informal notes during group investigations; the peer evaluation form and your presentation rubric; and the homework assignments.

XIII. Resources

Books:
Llewellyn, D. (2007). *Inquire Within: Implementing Inquiry-Based Science Standards in Grades 3-8*, 2nd Edition. Corwin Press.

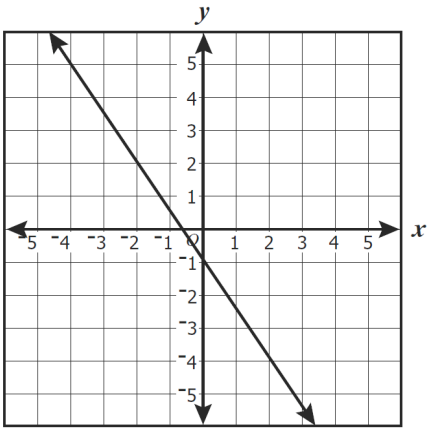
Websites:
National Security Agency. (n.d.). *Detective Slope – An Investigation of the Slopes of Lines and Shapes.* Retrieved August 20, 2014 from http://www.nsa.gov/academia/_files/collected_learning/high_school/geometry/detective_slope.pdf

Pre-assessment

1. What does slope represent?

2. List and draw and example of the 4 different types of slope.

3. What is the slope of the line graphed below? Explain your answer.



4. What is the slope of the line that passes through (-3, -5) and (4, -2)? Explain your answer.

Slope Scavenger Hunt

Your group needs to find at least 4 examples of “lines” around the school. Whenever possible have a student in the picture.

Pictures should include:

A Parallel Lines

B Perpendicular Lines (2 pictures)

Take at least one picture with perpendicular lines that are not vertical and horizontal.

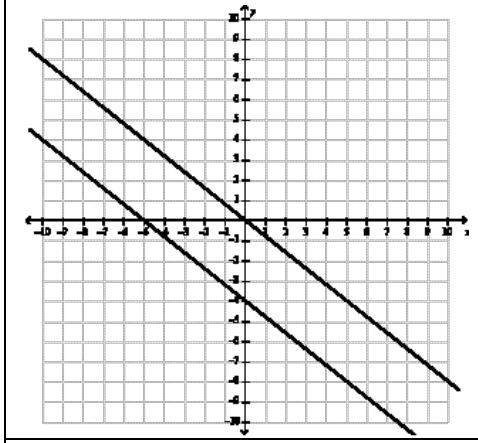
Take at least one picture of stairs.

C Intersecting Lines that are not perpendicular

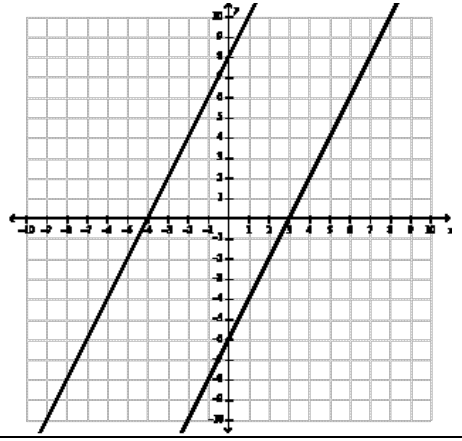
Slope Inquiry
Data Chart

	Slope of Line a	Slope of Line b	sum	product	difference	quotient	Relationship between the slope of line a and line b
Parallel Lines							
1.							
2.							
3.							
4.							
Perpendicular Lines							
5.							
6.							
7.							
8.							
Intersecting Lines (Not Perpendicular)							
9.							
10.							
11.							
12.							

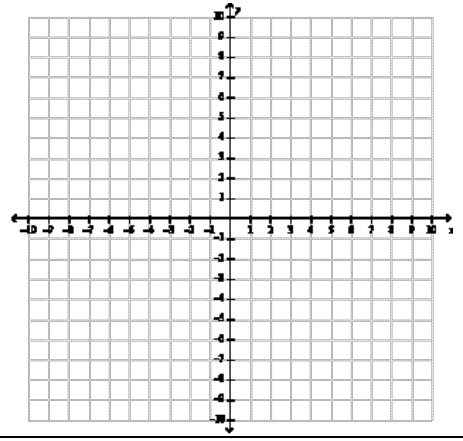
1. Parallel Lines



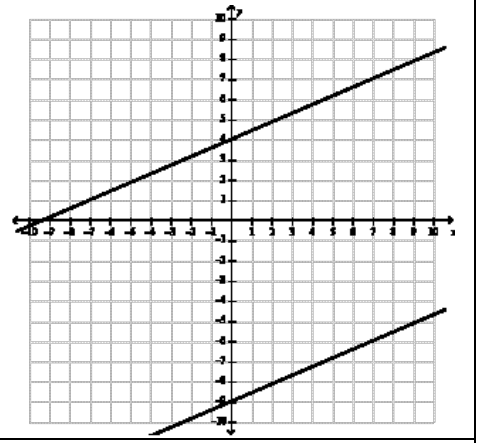
2. Parallel Lines



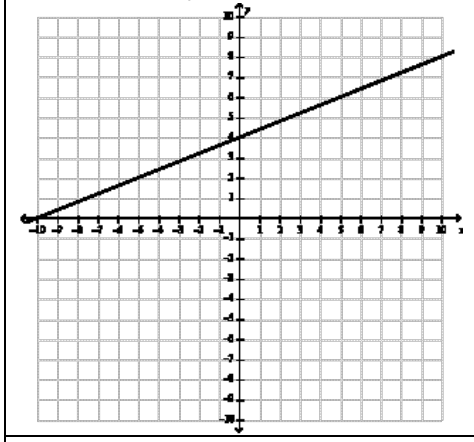
3. Parallel Lines



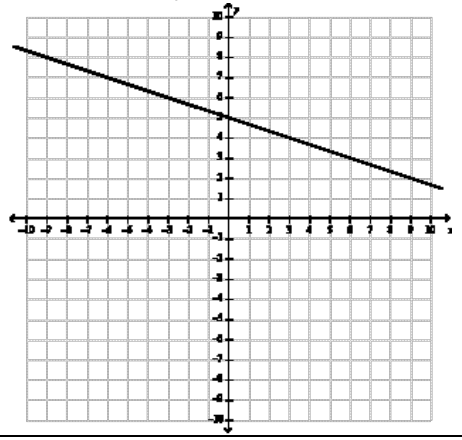
4. Parallel Lines



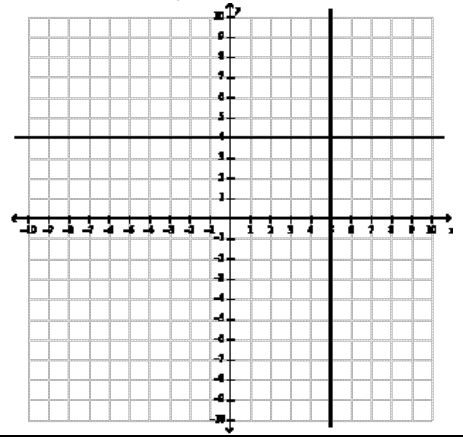
5. Perpendicular Lines



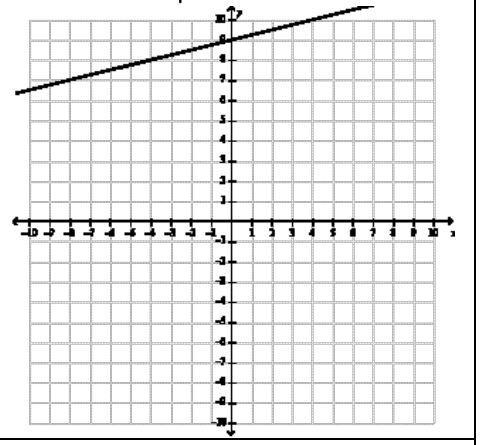
6. Perpendicular Lines



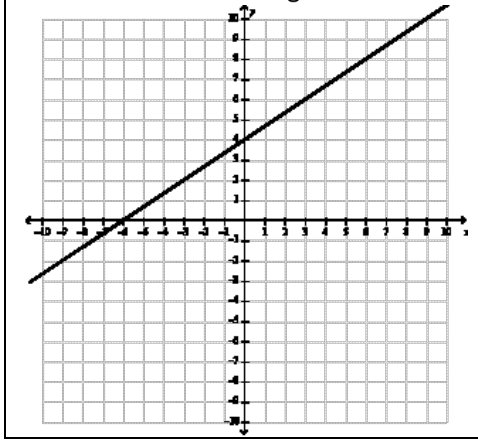
7. Perpendicular Lines



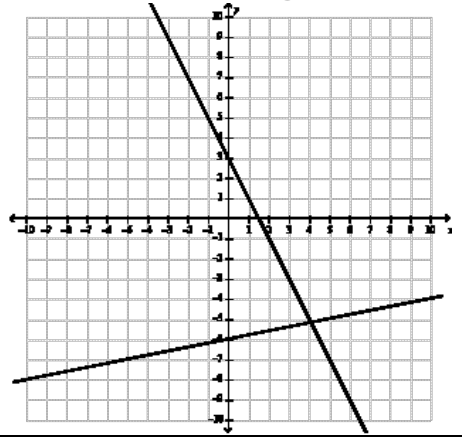
8. Perpendicular Lines



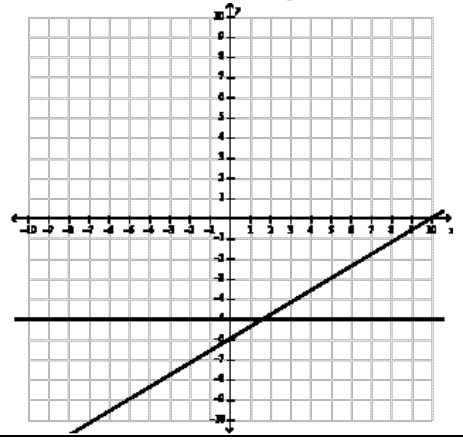
9. Intersecting Lines



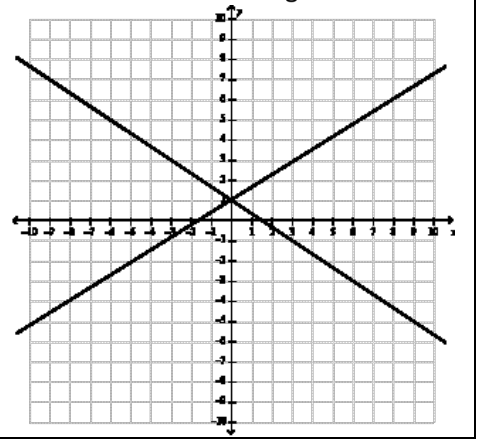
10. Intersecting Lines



11. Intersecting Lines



12. Intersecting Lines



Checklist for Discovering Line Relationship

Group	Parallel Lines	Perpendicular Lines	Intersecting Lines

Grid for pictures

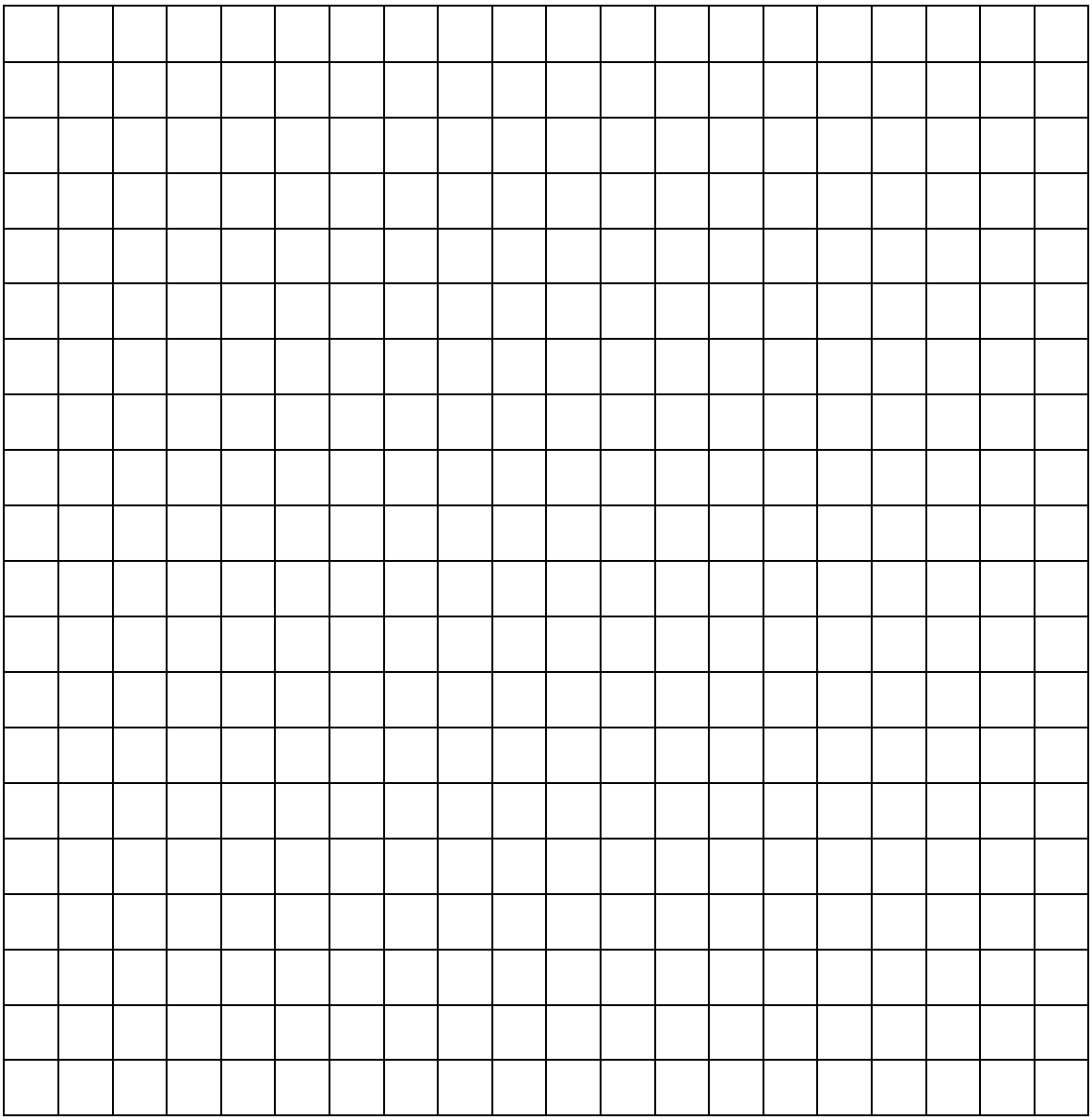
1. Insert picture.

2. Right click on the picture.

   Change text wrapping to behind text.

   Move the picture behind the grid.

3. Draw in your lines.



Parallel and Perpendicular Lines Peer Evaluation Form

Group Name: List at least two aspects of the presentation you liked. What could the group have done to make their presentation better? What did this group discover about the slopes of parallel lines? What did this group discover about the slopes of perpendicular lines?	**Group Name:** List at least two aspects of the presentation you liked. What could the group have done to make their presentation better? What did this group discover about the slopes of parallel lines? What did this group discover about the slopes of perpendicular lines?
Group Name: List at least two aspects of the presentation you liked. What could the group have done to make their presentation better? What did this group discover about the slopes of parallel lines? What did this group discover about the slopes of perpendicular lines?	**Group Name:** List at least two aspects of the presentation you liked. What could the group have done to make their presentation better? What did this group discover about the slopes of parallel lines? What did this group discover about the slopes of perpendicular lines?

Slopes of Parallel and Perpendicular Lines Presentation Rubric

Group Name: _____

_____ 1 point - Picture that shows parallel lines.

_____ 1 point - Picture that shows perpendicular lines.

_____ 1 point - Picture that shows intersecting lines that are not perpendicular.

_____ 1 Bonus point for an extra picture with perpendicular lines that are not horizontal and vertical.

_____ 12 points - 12 coordinate planes with lines used to develop the rules for proving lines are parallel, perpendicular, or neither.

_____ 3 points - Pictures with coordinate grids.

_____ 3 points - Slopes to go with the lines on their pictures.

_____ 12 points - Used the rules they discovered to prove their lines were parallel, perpendicular, or neither.

_____ 6 points – At least 3 sample questions for the class (Send these to teacher's email or have them on an 8 ½ x 11 piece of paper.)

_____ 5 points - Creative summary of their findings. (3 points will reflect the average presentation.)

_____ **out of 45 points total**

Slope Anchor Activity (optional homework)

Find the slope of each line.

1. $y = -6x - 2$

2. $y=-27x-5$

3. $y =25x+4$

4. $y = -x + 1$

5. $y=34x-3$

6. $x = 9$

7. $3x + 4y =8$

8. $4x + 5y = 9$

9. $y = -7$

10. $3(2x - 6) = 2y$

Slope Post-assessment

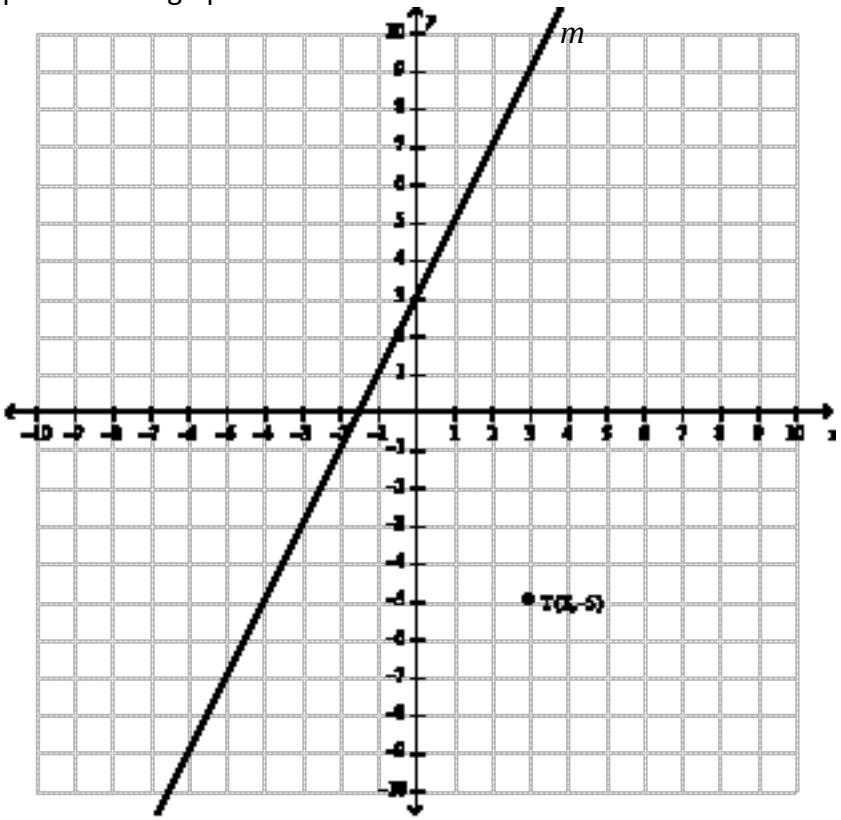
1. What is the slope of the line that passes through (2, -6) and (-4, -2)? Explain your answer.

2. How can you use the slopes of two lines to prove they are parallel?

3. How can you use the slopes of two lines to prove they are perpendicular?

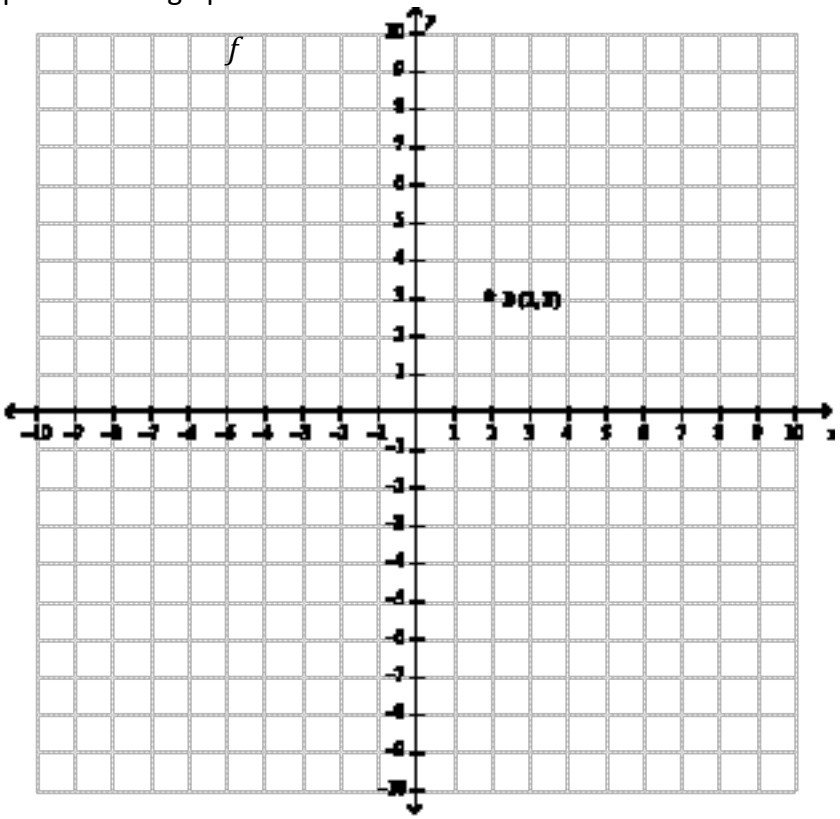
4. Line *m* contains points (1, 5) and (-2, -1)

Plot a point other than point T with integral coordinates that lies on a line that is parallel to *m* and passes through point T.



5. Line *f* contains points (-3, 4) and (1, -8)

Plot a point other than point B with integral coordinates that lies on a line that is perpendicular to *f* and passes through point B.



6. Is it possible for two lines with positive slopes to be perpendicular? Explain your answer.

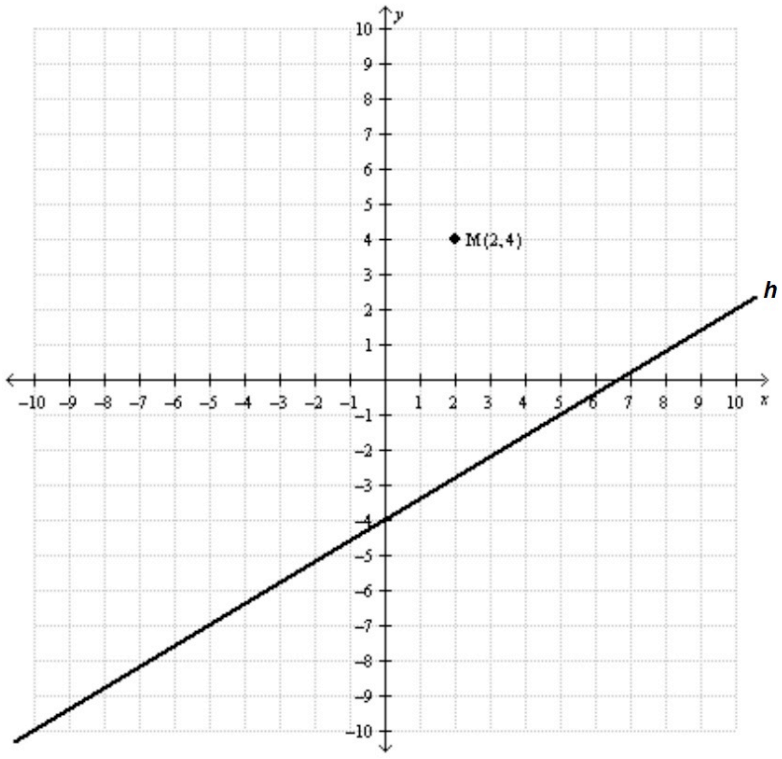
7. Line *g* passes through points (-1, -6) and (1, 2). Line *h* passes through points (-4, 5) and (4, 3). Are lines *g* and *h* parallel, perpendicular, or neither? Explain your answer.

8. Are the lines represented by the equations y = 2x – 3 and y = -2x – 3 parallel, perpendicular, or neither? Explain your answer.

9. If a line has a slope of -4/3, what would the slope of its perpendicular line be?

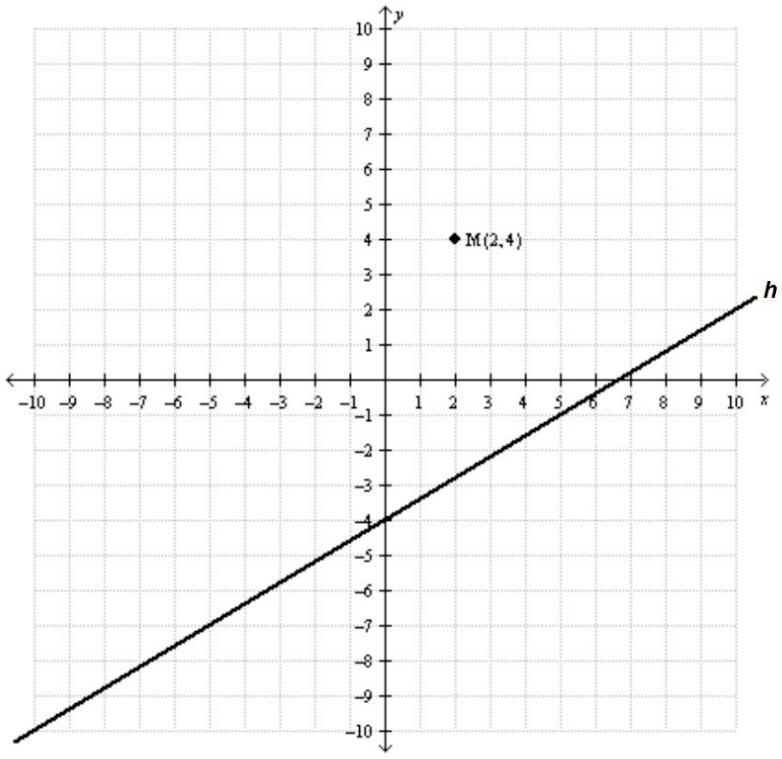
Line *h* contains the points (0, -4) and (5, -1).

Plot a point other than point M with integral coordinates that lies on a line that is parallel to *h* and passes through point M.



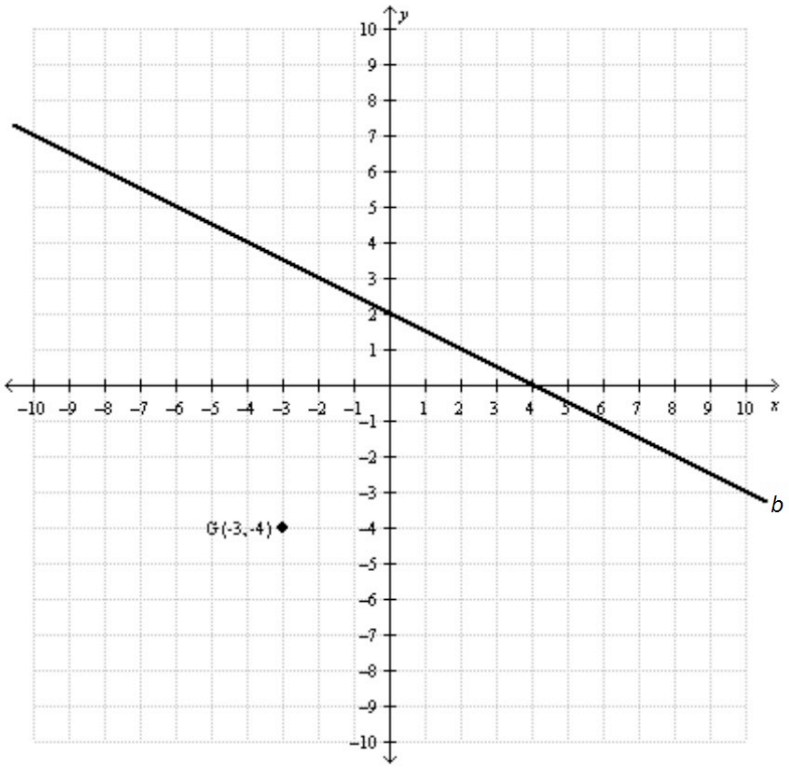
Line *h* contains the points (0, -4) and (5, -1).

Plot a point other than point M with integral coordinates that lies on a line that is perpendicular to *h* and passes through point M.



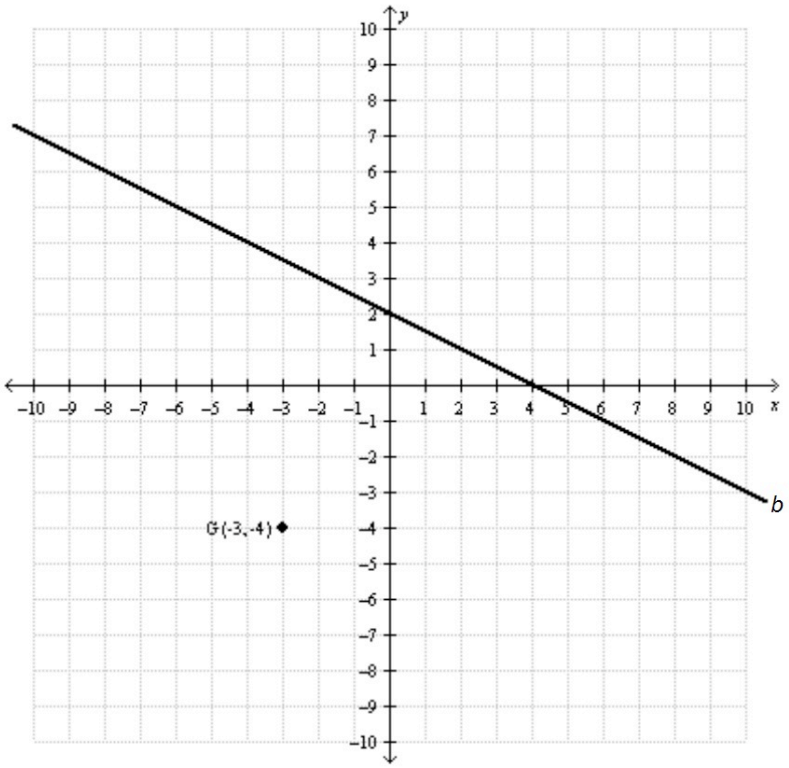
Line *b* contains the points (-4, 4) and (6, -1).

Plot a point other than point G with integral coordinates that lies on a line that is parallel to *b* and passes through point G.



Line *b* contains the points (-4, 4) and (6, -1).

Plot a point other than point G with integral coordinates that lies on a line that is perpendicular to *b* and passes through point G.



Select all of the equations that are parallel to the line y = 2x.

$y = 2x - 4$

$y = -\frac{1}{2}x$

$y = -2x$

$3y = 6x + 4$

$y = \frac{1}{2}x$

$3y = -6x + 2$

Select all of the equations that are parallel to the line y = 3x + 2.

$y = 3x$

$y = -\frac{1}{3}x$

$y = -3x + 2$

$3y = 9x + 7$

$y = \frac{1}{3}x$

$-3y = x + 6$